

## The Doppler Equation

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Knowledge of ultrasound physics and instrumentation enables the ultrasound user to optimally employ available echocardiographic modalities. The purpose of this and subsequent reviews is to discuss the areas of ultrasound physics and instrumentation that are particularly significant to the clinician. We will begin with a discussion of the Doppler equation.

Doppler echocardiography is used to measure blood flow (continuous wave and pulsed wave Doppler) and cardiac tissue (tissue Doppler imaging) velocities. When either the source or the recipient of a traveling wave is moving, then the wave recipient perceives a frequency that is different from the emitted frequency.<sup>1</sup> This change in perceived frequency is known as the Doppler effect. As applied to Doppler echocardiography, an ultrasound transducer that is electrically stimulated either continuously (continuous wave) or in bursts (pulsed wave) is aimed directly at a stream of flowing blood or moving cardiac tissue. When the resulting emitted ultrasound signal strikes the red blood cells or tissue, a small proportion of its energy is reflected back to the ultrasound probe. If the target is traveling toward the transducer, the frequency of the reflected ultrasound is greater than the emitted frequency. The increase in frequency is termed a positive Doppler shift (Fig. 1). If the blood or tissue is moving away from the transducer, the reflected frequency is lower (a negative Doppler shift). The difference between the emitted ( $f_0$ ) and the reflected ( $f_r$ ) ultrasound frequencies, termed the

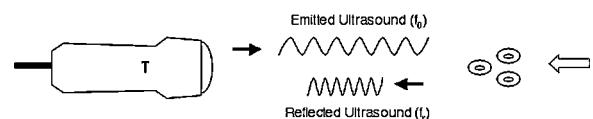
Doppler shift frequency ( $f_D$ ), is determined according to the following equation:

$$f_D = (f_r - f_0) = 2f_0v(\cos \theta)/c \quad (1)$$

where  $v$  is the red blood cell or tissue velocity,  $\cos \theta$  is the cosine of the angle between the path of the ultrasound beam and that of blood flow or tissue, and  $c$  is the average speed of ultrasound in soft tissue (1540 m/sec). There are actually two Doppler shifts that occur (hence the number 2 in the numerator), for the moving target first acts as the sound recipient, and then as the source as ultrasound strikes it and reflects back toward the transducer. By rearranging the equation, one may solve for velocity:

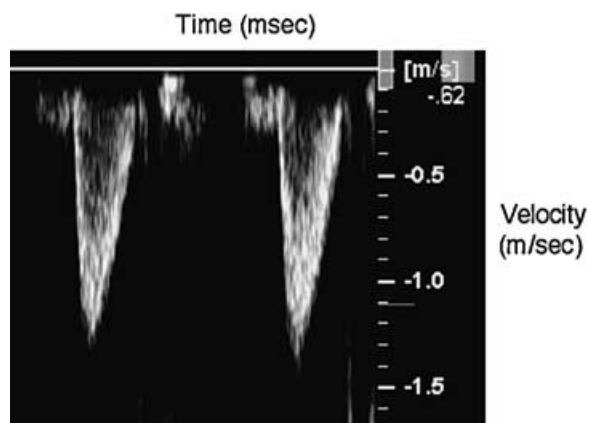
$$v = cf_D/2f_0(\cos \theta) \quad (2)$$

With regard to the Doppler equation, there are several points that require explanation. To begin, the presence of  $f_0$  in the Doppler equation suggests that there is one emitted ultrasound frequency. Although there is a primary transmitted frequency termed the center frequency, which is typically ~2 MHz in commercial ultrasound machines, additional frequencies are present in the emitted beam or pulse. As a result, there are multiple reflected frequencies as well. The number of reflected frequencies is also influenced by the number of different ultrasound reflector velocities present.



**Figure 1.** Representation of a positive Doppler shift. When a traveling wave of ultrasound strikes red blood cells that are moving toward the ultrasound transducer (T), the frequency of the reflected ultrasound wave ( $f_r$ ) is increased above that of the emitted wave ( $f_0$ ).

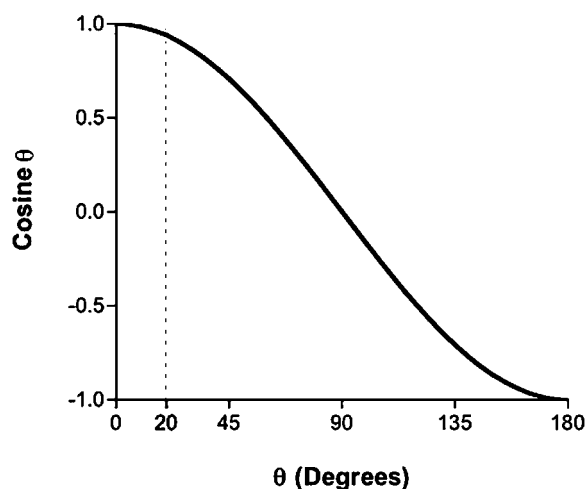
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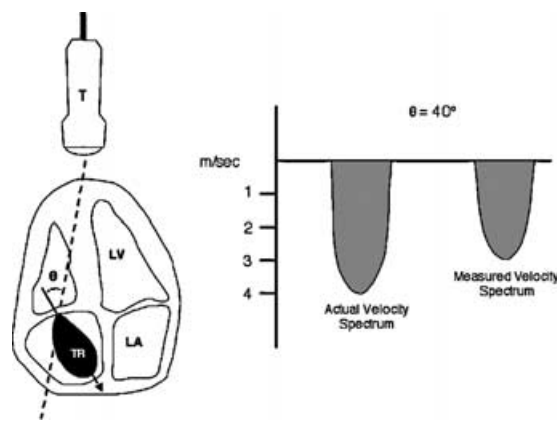
**Figure 2.** Velocity spectra of normal systolic blood flow in the left ventricular outflow tract.

Thus, at any point of time there is a range of Doppler shift frequencies. Using Equation (2), Doppler shift frequencies are converted to velocities and displayed as a velocity spectrum (Fig. 2), which is the basis for the term “spectral Doppler.” The center frequency can be adjusted by the user and its significance will become apparent in a later discussion on aliasing.

An important component of the Doppler equation is the cosine  $\theta$  term. Because of its presence in the numerator in Equation (1), as the angle between the axis of the ultrasound beam and the direction of blood flow increases ( $\theta > 0^\circ$ ), then cosine  $\theta$  becomes less than 1 (Fig. 3) and the measured Doppler shift frequency is reduced. As a result, the calculated velocity is also reduced (Fig. 4). This occurs be-



**Figure 3.** Cosine of angles 0–180°. At  $\theta = 20^\circ$ , cosine  $\theta$  is equal to  $\sim 0.94$ , leading to 6% underestimation of peak blood flow velocity.



**Figure 4.** Peak velocity of tricuspid regurgitation is underestimated when the Doppler cursor (dashed line) is not aligned with the axis (arrow) of the regurgitant flow. LA = left atrium; LV = left ventricle; TR = tricuspid regurgitant backflow;  $\theta$  = angle between axes of ultrasound beam and blood flow.

cause the ultrasound machine assumes that  $\theta = 0^\circ$ ; the cosine of 0 is 1, so cosine  $\theta$  effectively drops out of Equation (2). Therefore, in order to accurately measure peak velocities of blood flow, the ultrasound beam (Doppler cursor) should be oriented as close as possible to the direction of blood flow. However, an angle between the ultrasound beam and the axis of blood flow of up to  $20^\circ$  is generally acceptable, for at  $\theta = 20^\circ$ , there is only a 6% error in the measurement of peak blood flow velocity. An angle-correct function is present on all ultrasound machines and its cursor could potentially be directed along the apparent axis of blood flow, thereby telling the machine the value of  $\theta$ . However, blood flows in three dimensions, so its path cannot necessarily be approximated with the two-dimensional angle-correct cursor. Therefore, the American Society of Echocardiography does not recommend its use.<sup>2</sup>

In summary, the Doppler equation has been described. A major clinical ramification of this equation is the impact of its cosine  $\theta$  term, which influences the user’s ability to accurately measure peak blood flow velocities.

**References**

1. Weyman, AE: *Principles and Practice of Echocardiography, 2nd Ed.* Malvern, PA, Lea & Febiger, 1994, pp. 143–215.
2. Quinones, MA, Otto, CM, Stoddard, M, et al: Recommendations for quantification of Doppler echocardiography: A report from the Doppler Quantification Task Force of the Nomenclature and Standards Committee of the American Society of Echocardiography. *J Am Soc Echocardiogr* 2002;15:167–184.